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**Fracture Mechanics Final Exam**

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From the beam theory of cantilever maximum displacement, we have:

, where L is the length of the beam and  is maximum displacement

For the upper arm of the component, we have:

* Inertia: 
* Compliance:  (answer)

For the lower arm of the component, we have:

* Inertia: 
* Compliance:  (answer)

The compliance of the whole system is

 (answer)

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The energy release rate of the whole component system is



*  (answer)

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Under displacement control, the energy release rate is:



Assuming that the material has a flat R-curve, crack growth is unstable under displacement control because of  (answer)

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According to the formula in of fracture toughness:

 (answer)

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The aluminum plate is thin => The plane stress condition is assumed

Therefore, the plastic zone size under plane stress is:



If the plastic zone size  is roughly an order of magnitude smaller than the crack length (), we conclude that Linear Elastic Fracture Mechanics (LEFM) applies, and fracture will occur when . Since , LEFM can be applied here.

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The stress intensity factor when the fracture stress is increased to  becomes:



By principal of superposition, the fracture toughness with wire reinforcement becomes

, where negative sign of  indicates compression on the aluminum plate

* 

The stress intensity induced by the wire is:

, which is a reduced formula from  where 

* 

The force P needed to increase fracture stress to  is  (answer)

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The plate is thin and large => The plane stress condition is assumed. The energy release rate is

* 

The moment at which fracture will become unstable is when:

(I)  and (II) 

The first condition gives us:

 (I)

Whereas the second condition returns:

 =>  (II)

Combining two equations, we have the following equality:

 => 

* => 
* . Replace  into the equation
*  => 

Therefore, the amount of stable crack growth is  (answer)

We can substitute the stable crack growth into the second equation to obtain the critical stress



* 

Therefore, the stress at which unstable fracture occurs is  (answer)

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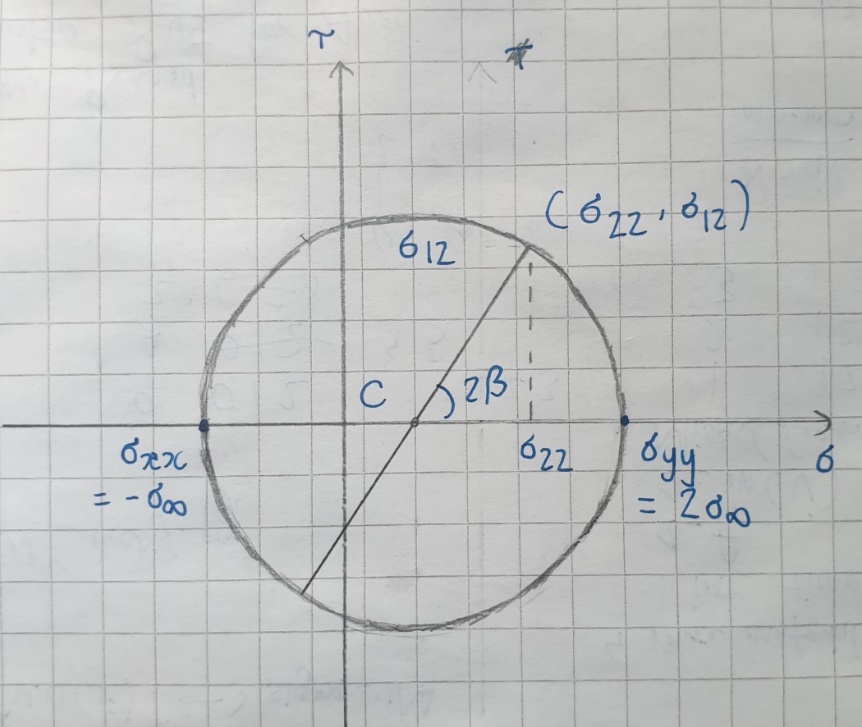
The stress field in the global reference frame is:



The first and second points on the Mohr circle are:  and 

* The two points are  and 

The Mohr’s circle for this stress field is:



where the center C and radius R of the circle are:  and 

The stresses  and  in the local reference frame are given by:


Finally, the stress intensity factors as a function of and  are given by:

 (answer)

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To find the angle of crack propagation, we set , and this gives:



* 

We are also given the information that 

* . Let . Replace this into the equation
* 

We can apply these two identities to simplify the equation

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* 
* 
* 
* 

Plot the trigonometric function in MATLAB and we can observe that it has six roots in . Now we can use numerical solutions to find the six roots of the function

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The correct angle  is the one corresponding to the maximum 



We are also given the information that , so we need to maximize this quantity



Plugging the solutions above:

=>  => 

=>  => 

=>  => 

Therefore, it shows that  corresponds to a maximum . Therefore, the crack will propagate along (answer)